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MEASUREMENT OF FLOW MAGNITUDE AND DIRECTION BY HOT WIRE ANEMOMETER

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ABSTRACT

A technique is developed for the measurement of mean velocity direction and magnitude and the turbulence quantities in incompressible fluid flow within a plane, when the flow direction is known only approximately within the plane.

A low degree polynomial is fit to the calibration data by least squares, in the form: square root of velocity versus square of voltage. Thereby known approximate adherance to King's law is used to advantage, yet accounting for small deviations from the law.

A normal-component cooling model accounts for probe directional sensitivity. Calibration is performed at flow directions near those expected in the measurement situation, and the model is used to find a calibrated wire reference angle. Data taken at several flow angles revealed that the normal cooling model worked well in reducing the calibration data to a single curve.

Measurements of turbulent shear stress in fully developed pipe flow gave excellent agreement with a theoretical prediction of total shear distribution. The techniques developed were also used to measure the velocity field of an ejector.

MEASUREMENT OF FLOW MAGNITUDE AND DIRECTION BY HOT WIRE ANEMOMETER

INTRODUCTION

A technique is developed for the measurement of mean velocity direction and magnitude and the turbulence quantities in incompressible fluid flow within a plane, when the flow direction is known only approximately within the plane. This situation may arise, for example, in the study of the development region of swirl-free axisymmetric pipe flow or in the study of the velocity field of a jet flow.

The development of the method is restricted to a low level of turbulence such that the anemometer response to velocity perturbations is linear, i.e., for "small" velocity perturbations. Use is made of the knowledge of approximate King's law behavior and of an assumed normal-component cooling effect.

The technique is checked by comparing measured results with known behavior in fully developed pipe flow. This case does constitute a significant simplification of procedure in that the flow direction is known to be parallel to the axis of the duct. Data were also taken in the mixing region of a ducted axisymmetric jet, where the flow direction is not known; and some sample results are presented.

ANALYSIS

Quantities Measured

The flow quantities to be measured with respect to reference axes in a plane are mean velocity magnitude, $\overline{U}_R$, mean direction of flow δ, longitudinal turbulence velocity $\left\{\overline{u_R^2}\right\}^{1/2}$, transverse turbulence velocity $\left\{\overline{v_R^2}\right\}^{1/2}$, and Reynolds stress $\overline{u_R v_R}$.

Fundamentals

King's law for an anemometer may be written

$$E^2 - E_o^2 \propto U_{eff}^{1/2}$$

where E is the voltage of the wire, and E_o is the voltage at $U_{eff} = 0$, and U_{eff} is the effective cooling velocity. This relation is developed from considerations of the wire energy balance including convective cooling; it is possible to evaluate the proportionality from the analysis. However, in practice a calibration of the wire is developed by measuring E in a known flow. Since the relation of E^2 versus $U^{1/2}$ is nearly a linear one, the calibration data may be accurately curve-fit by a polynomial of low degree, e.g.,

$$E_c^2 = f\left(U_c^{1/2}\right) = a_o + a_1U_c^{1/2} + a_2U_c + a_3U_c^{3/2} \tag{1}$$

or

$$U_c^{1/2} = g\left(E_c^2\right) = b_o + b_1E_c^2 + b_2E_c^4 + b_3E_c^6 \tag{2}$$

These calibration curves may then be employed in determining wire sensitivities to turbulence fluctuations and in determining mean velocities.

Use of a Normal-Sensor Probe

Assuming that the turbulence fluctuations are "small" and that the calibration relation, developed in a low turbulence flow, applies instantaneously in the flow, one can develop working relations. For a sensor which is oriented normal to the mean velocity, Fig. 1,

$$\overline{U} = f_n(\overline{E}) \tag{3}$$

and

$$\overline{e^2} = S_u^2\overline{u^2} \tag{4}$$

where $S_u = \partial E_c/\partial U_c|_{\overline{U}}$. Thus once the wire has been calibrated (E_c versus U_c determined) the values of $\overline{U}$ and $\overline{u^2}$ may be determined by measuring $\overline{E}$ and $\overline{e^2}$.

Use of an Inclined-Sensor Probe

For a probe lying in the xy plane and inclined to the mean velocity (assuming x corresponds to the direction of the mean velocity $\overline{U}$), Fig. 2, the following relations apply:

$$E = f(U, \varphi) \tag{5}$$

$$e = S_u u + S_v v \tag{6}$$

$$S_u = \left.\frac{\partial E}{\partial U}\right|_{\overline{U}} \tag{7}$$

$$S_v = \frac{1}{U}\left.\frac{\partial E}{\partial \varphi}\right|_{\overline{U}, \varphi} \tag{8}$$

The relation f is fixed by calibration and may be expressed

$$E_c = f(U_c, \varphi_c) \tag{9}$$

In the general situation the probe is held so that the sensor has a fixed orientation with respect to reference coordinates x_R, y_R, z_R; these coordinates are set by the geometry of the situation (see fig 3) and may correspond, for example, to the axial, radial, and tangential directions in a cylindrical duct. For the present purpose the x, y and the x_R, y_R planes are coincident, however the angle φ is unknown. In the measurement situation the unknowns are $\overline{U}$, φ, $\overline{u^2}$, $\overline{v^2}$, and $\overline{uv}$, with respect to a set of flow coordinates arranged such that the direction of mean velocity vector and the x coordinate direction coincide. It is ultimately desired to know the velocity and turbulence quantities relative to the coordinates x_R, y_R, and z_R which are fixed by the geometry. Therefore δ, the angle between U(x) and x_R, must be found. The unknowns are U_R, δ, $\overline{u_R^2}$, $\overline{v_R^2}$, and $\overline{u_R v_R}$ and at least 5 measurements are required to determine them.

To see how these measurements are accomplished, consider the sensor arranged at two different orientations relative to the flow, in the plane of interest, Fig. 3. The angle φ_o is fixed by geometric constraints,

i.e., the sensor may be held by a probe body lying along the y_R axis; and since φ_o is thereby fixed, δ becomes the variable of interest in the calibration and use of the probe.

The calibration relations may be stated:

$$E_{c1} = f_1(U_c, \delta) \tag{10}$$

$$E_{c2} = f_2(U_c, \delta) \tag{11}$$

For purposes of determining U and δ, these can be written:

$$U_c = g_1(E_{c1}),\ \delta \text{ parameter} \tag{12}$$

$$U_c = g_2(E_{c2}),\ \delta \text{ parameter} \tag{13}$$

These relations are established by setting up a table of E_c, U_c, δ in a flow whose direction and magnitude are controlled. However, as it is, the table is not very helpful in determining $\overline{U}$ and δ when $\overline{E}_1$ and $\overline{E}_2$ are measured in a flow of unknown velocity.

The use of the calibration data would be facilitated if the angular dependence could be handled in equation form. It is well known that a hot wire is cooled primarily by the component of velocity normal to itself. By calibrating the probe at values of φ, i.e., δ, near the values expected in the measurement situation, effects due to the velocity component along the wire may be accounted for in the mean. Therefore

$$\begin{aligned} U_{eff} &= U \sin \varphi \\ &= U \sin (\varphi_o \pm \delta) \end{aligned} \tag{14}$$

with the plus used in orientation 1 and the minus used in orientation 2.

With the above assumption, one may write

$$\left\{U \sin \varphi\right\}^{1/2} = g(E^2) = b_o + b_1 E^2 + b_2 E^4 + \ldots\ldots \tag{15}$$

and

$$E^2 = f\left\{U_c \sin \varphi\right\}^{1/2} \tag{16}$$

The above formulations have been checked by calibrating the wire in orientation one in a known flow, perturbing its position by amount δ about angle φ_o. A sample of the result may be seen in Fig. 4. For the range of δ's chosen the data do indeed collapse to a single curve. The same procedure must be carried out for orientation two (no figure shown); the results will be the calibration relations

$$\left\{U \sin(\varphi_o + \delta)\right\}^{1/2} = g_1\left(\overline{E}_1^2\right) \tag{17}$$

$$\left\{U \sin(\varphi_o - \delta)\right\}^{1/2} = g_2\left(\overline{E}_2^2\right) \tag{18}$$

These relations are then used, once $\overline{E}_1$ and $\overline{E}_2$ are measured in the measurement situation, to determine $\overline{U}_R$, $\overline{V}_R$. Using appropriate trigonometric identities

$$U_R = U \cos\delta = \frac{g_1^2\left(\overline{E}_1^2\right) + g_2^2\left(\overline{E}_2^2\right)}{2 \sin\varphi_o} \tag{19}$$

$$V_R = U \sin\delta = \frac{g_1^2\left(\overline{E}_1^2\right) - g_2^2\left(\overline{E}_2^2\right)}{2 \cos\varphi_o} \tag{20}$$

Therefore

$$\delta = \tan^{-1}\frac{\overline{V}_R}{\overline{U}_R} \tag{21}$$

$$\varphi_1 = \varphi_o + \delta \tag{22}$$

$$\varphi_2 = \varphi_o - \delta \tag{23}$$

Thereby the mean velocities are determined, assuming φ_o is known.

One may assume $\varphi_o = 45^o$, as probe manufacturers attempt to accomplish, or one may determine φ_o by using the assumed normal-component cooling behavior:

$$\left\{U \sin (\varphi_o + \delta)\right\}^{1/2} = g(E^2) \tag{24}$$

In the calibration rig U and δ are known and E is measured. In orientation one or two, one may set two known values of $\delta(\delta_A$ and $\delta_B)$ and control U in the two cases so that

$$E_A = E_B \tag{25}$$

For the case $\delta_A = 0$, $\delta_B \neq 0$, using an appropriate trigonometric identity:

$$U_A \sin \varphi_o = g^2(E_A^2) = g^2(E_B^2) = U_B (\sin \varphi_o \cos \delta_B + \cos \varphi_o \sin \delta_B) \tag{26}$$

thus

$$\tan \varphi_o = \frac{U_B \sin \delta_B}{U_A - U_B \cos \delta_B} \tag{27}$$

In practice these steps may be carried out several times, and an average value for φ_o may be taken.

It remains yet to determine the turbulence quantities. Equation leads to the following for orientations one and two:

$$\overline{e_1^2} = S_{u1}^2 \overline{u^2} + 2 S_{u1} S_{v1} \overline{uv} + S_{v1}^2 \overline{v^2} \tag{28}$$

$$\overline{e_2^2} = S_{u2}^2 \overline{u^2} + 2 S_{u2} S_{v2} \overline{uv} + S_{v2}^2 \overline{v^2} \tag{29}$$

which may be solved for $\overline{uv}$ and $\overline{v^2}$:

$$\overline{uv} = \frac{S_{v2}^2 \overline{e_1^2} - S_{v1}^2 \overline{e_2^2} - \left(S_{u1}^2 S_{v2}^2 - S_{v1}^2 S_{u2}^2\right) \overline{u^2}}{2 S_{v1} S_{v2} \left(S_{u1} S_{v2} - S_{v1} S_{u2}\right)} \tag{30}$$

$$\overline{v^2} = \frac{\overline{e_1^2}}{S_{v1}^2} - \left(\frac{S_{u1}}{S_{v1}}\right)^2 \overline{u^2} - 2 \frac{S_{u1}}{S_{v1}} \overline{uv} \tag{31}$$

These relations, however, require that $\overline{u^2}$ be given; this requires an additional measurement. To determine $\overline{u^2}$ a separate wire may be placed in the flow at the same location such that $\overline{U}$ is always normal to the wire, thus sensitive to $\overline{u^2}$. This implies aligning the wire along the z axis; the procedures for a normal wire may then be applied.

Finally the turbulence components with respect to the reference coordinates are:

$$\overline{u_R^2} = \overline{u^2} \cos^2\delta + \overline{v^2} \sin^2\delta - 2\,\overline{uv} \cos\delta \sin\delta \tag{32}$$

$$\overline{v_R^2} = \overline{u^2} \sin^2\delta + \overline{v^2} \cos^2\delta + 2\,\overline{uv} \cos\delta \sin\delta \tag{33}$$

$$\overline{u_R v_R} = (\overline{u^2} - \overline{v^2}) \cos\delta \sin\delta + (\cos^2\delta - \sin^2\delta)\,\overline{uv} \tag{34}$$

The wire sensitivities for each of orientations one and two are determined as follows:

$$E_c^2 = f(\arg) \tag{35}$$

$$\arg = (U_c \sin\varphi)^{1/2} \tag{36}$$

Here f represents the curve-fit of the calibration data

$$E_c^2 = f(\arg) = a + a_1 \arg + a_2 \arg^2 + a_3 \arg^3 + \ldots\ldots \tag{37}$$

From these

$$S_u = \left.\frac{\partial E}{\partial U}\right|_{\overline{U},\ \varphi} = \left\{\frac{df}{darg}\ \frac{\sin\varphi}{4\,E\,(U\sin\varphi)^{1/2}}\right\}\Bigg|_{\overline{U},\ \varphi,\ E} \tag{38}$$

$$S_v = \left.\frac{1}{U}\frac{\partial E}{\partial\varphi}\right|_{\overline{U},\ \varphi} = \left\{\frac{df}{darg}\ \frac{\cos\varphi}{4\,E\,(U\sin\varphi)^{1/2}}\right\}\Bigg|_{\overline{U},\ \varphi,\ E} \tag{39}$$

where df/darg is found by differentiating the least squares curve-fit of the calibration data.

RESULTS

In order to evaluate the techniques which have been developed, measurements of $\overline{uv}$ were made in fully developed pipe flow and the results were compared with a theroetical prediction of the total shear stress distribution for such a flow. Recall that in turbulent flow the turbulent shear stress $\overline{uv}$ is virtually equal to the total shear stress everywhere in the pipe except near the wall where a laminar viscous shear layer exists. While the fully developed flow $(\delta = 0)$ permitted significant simplifications, the results provide a reasonable check of the techniques.

As seen in Fig. 5 the $\overline{uv}$ results are in good agreement with the theoretical prediction. In the figure τ represents the shear stress, ρ represents the air density, and U_τ represents the wall shear velocity found from measurement of pressure loss per unit length in the pipe, y is the distance from the wall and R is the pipe radius.

Additionally Fig. 4 has already shown the effectiveness (in the δ range considered) of collapsing the calibration curves via the assumed normal cooling model.

Results of Measurements in a Ducted Jet

Some sample results of mean velocity magnitude and direction and turbulent stress in the mixing region of an axisymmetric ducted air jet are presented in Fig. 6 and 7. The reported variables are defined relative to cylindrical coordinates x, r, and θ, which correspond to x_R, y_R, and z_R respectively from the section on the use of an inclined-sensor probe. Thus U_R is the component of mean velocity in the axial direction; delta is the angle between the mean velocity vector and the

axial direction (positive for outward directed vector); and $\overline{u_R v_R}$ is the turbulent Reynold shear stress corresponding to axial-radial coordinates.

The results shown are characteristic of jet mixing, with a zone of high radial mean velocity gradient between the inner and outer flows. The flow direction plot shows the outward spread of the central high speed flow and the inward entrainment of the outer flow into the mixing zone. Finally the plot of $\overline{u_R v_R}$, Fig. 7, shows the high turbulent shear stress resulting from the large velocity gradient between the inner and outer flows and the very low shear stress in the potential outer flow.

These results have been selected to demonstrate the practical application of the hot wire techniques. For a complete report of the ducted jet results, see Ref. 1.

SUMMARY OF RESULTS

Techniques have been presented for the measurement of mean velocity magnitude and direction and of turbulence quantities. The results were:

1. Good agreement was found between the data and a theoretical prediction of shear stress in fully developed pipe flow. The pipe flow test did comprise a significant simplification in that the mean flow direction was known throughout.

2. The use of the normal-component cooling model was effective in accounting for the directional sensitivity of the probe in the range of angles considered. The model was also used to establish the reference angle of the probe.

CONCLUDING REMARKS

A procedure has been developed for determining, within a plane, time mean velocity magnitude and direction and turbulence quantities. It is required that the flow direction be approximately known within this plane in order to effectively calibrate the probes.

Two probes are required for the measurements: one which can be placed normal to the plane at all points of interest, and a second one which is placed in the plane at first one and then a second inclination relative to the flow.

The limitations of the method are that:

1. The probe configurations must be achievable, and obviously one must be able to reach the "same point" with the two probes.

2. The flow field must be steady in the mean, since the measurements are not taken simultaneously.

3. Due to the assumption of linear voltage response to perturbations of velocity, velocity fluctuations must be "small".

4. In the region of interest, the sine law cooling assumption must adequately describe the directional effect of the flow.

5. The calibration of the probe in a low turbulence flow applies instantaneously in the flow.

6. All second order effects (e.g., wire length) have been neglected.

Some notable benefits of the method are that:

1. Subject to the indicated restriction, the directional dependence of the calibration is handled in equation form.

2. A digital computer may be readily applied for calibration and data reduction, and some on-line arrangement could be possible.

3. The curve fit is not forced to represent the calibration data with a single exponent, which would depend on the range of velocity.

4. The reference angle of the probe φ_0 is determined experimentally, consistent with the assumed normal component cooling model.

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J. O. Hinze, Turbulence (McGraw-Hill Book Co., Inc., New York, 1959).

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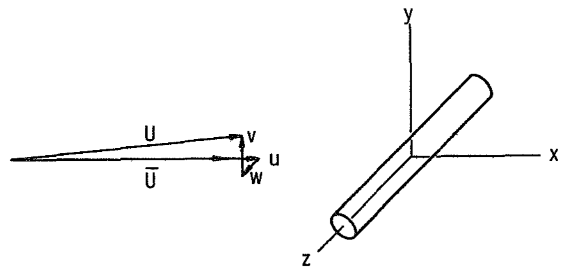

Figure 1. - Normal sensor in the x-z plane.

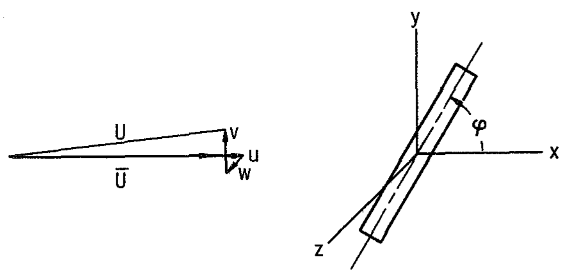

Figure 2. - Inclined sensor in the x-y plane.

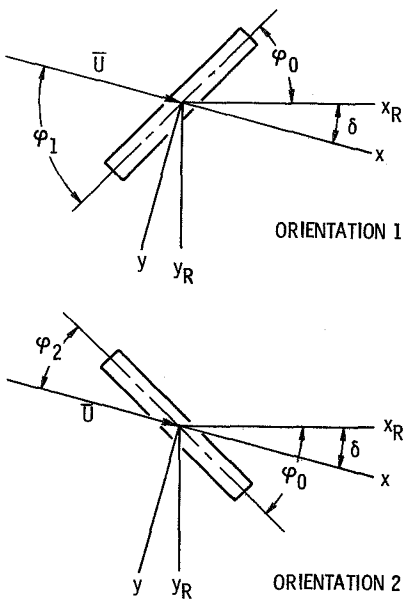

Figure 3. - Inclined sensor at two orientations.

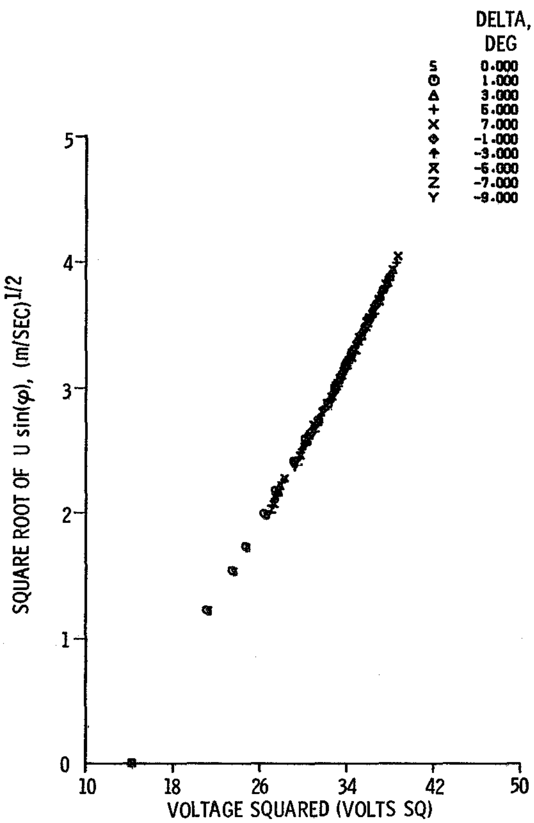

Figure 4. - Inclined hot wire (pos 1) calibration.

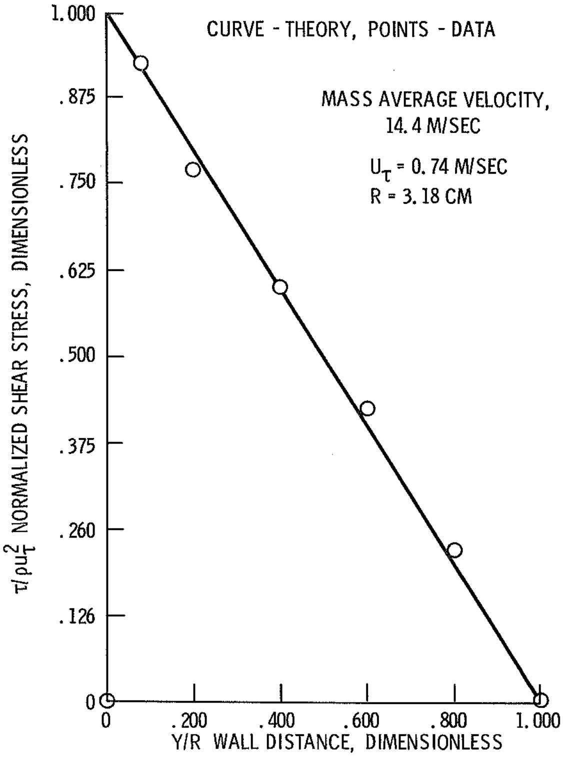

Figure 5. - Shear stress as a function of distance to wall in fully developed pipe flow.

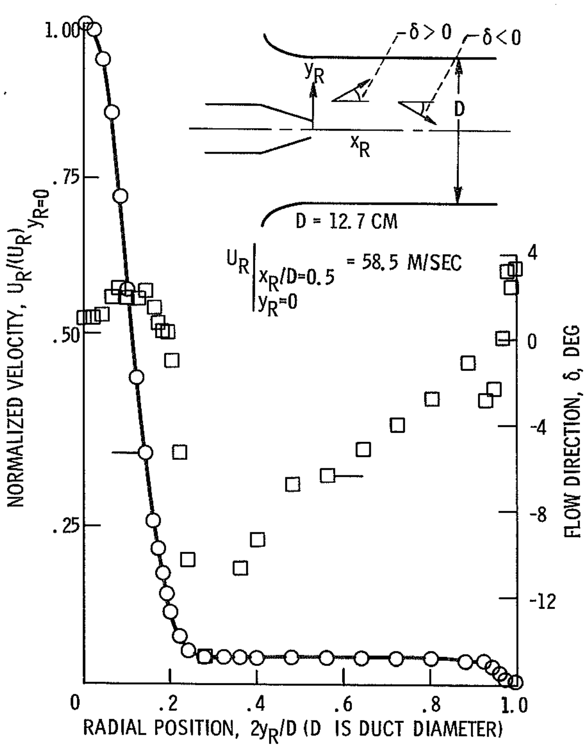

Figure 6. - Mean velocity magnitude and direction as a function of radial position in a ducted jet at x_R/D = 0.5.

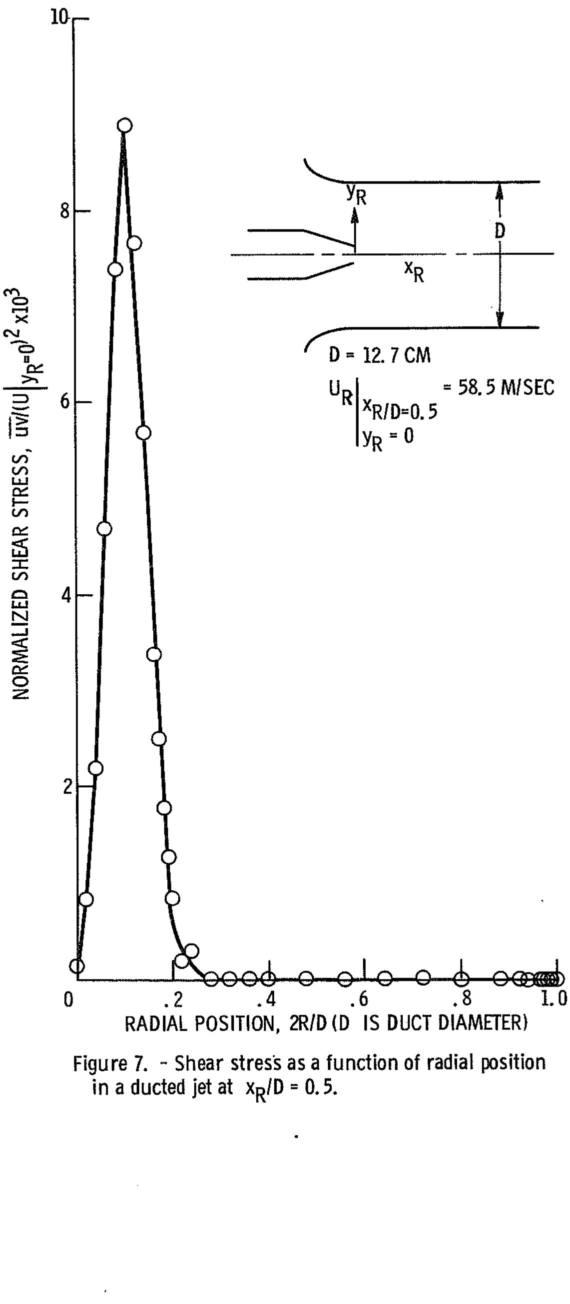

Figure 7. - Shear stress as a function of radial position in a ducted jet at x_R/D = 0.5.